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**Algorithm 19.5 Proposal distribution for collapsed Metropolis-Hastings over data completions**


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Procedure Proposal-Distribution (
   $\mathcal{G}$ , // Bayesian network structure over  $X_1, \dots, X_n$ 
   $\mathcal{D}$  // completed data set
   $X_i$  // A variable to sample
)
1  $\theta \leftarrow \text{Estimate-Parameters}(\mathcal{D}, \mathcal{G})$ 
2  $\mathcal{D}' \leftarrow \mathcal{D}$ 
3 for each  $m = 1 \dots M$ 
4   Sample  $x'_i[m]$  from  $P(X_i[m] \mid \mathbf{x}_{-i}[m], \theta)$ 
5 return  $\mathcal{D}'$ 

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**Exercise 19.24**

Prove theorem 19.8.

**Exercise 19.25\***

Prove theorem 19.10. Hint: Use the proof of theorem 19.5.

**Exercise 19.26**

Consider learning structure in the setting discussed in section 19.4.3.3. Describe a data set  $\mathcal{D}$  and parameters for a network where  $X_1$  and  $C$  are independent, yet the expected sufficient statistics  $\bar{M}[X_1, C]$  show dependency between  $X_1$  and  $C$ .

**Exercise 19.27**

Consider using the structural EM algorithm to learn the structure associated with a hidden variable  $H$ ; all other variables are fully observed. Assume that we start our learning process by performing an E-step in a network where  $H$  is not connected to any of  $X_1, \dots, X_n$ . Show that, for any initial parameter assignment to  $P(H)$ , the SEM algorithm will not connect  $H$  to the rest of the variables in the network.

**Exercise 19.28**

Consider the task of learning a model involving a binary-valued hidden variable  $H$  using the EM algorithm. Assume that we initialize the EM algorithm using parameters that are symmetric in the two values of  $H$ ; that is, for any variable  $X_i$  that has  $H$  as a parent, we have  $P(X_i \mid \mathbf{U}_i, h^0) = P(X_i \mid \mathbf{U}_i, h^1)$ . Show that, with this initialization, the model will remain symmetric in the two values of  $H$ , over all EM iterations.

**Exercise 19.29**

Derive the sampling update equations for the partition-based Gibbs sampling of equation (19.17) and equation (19.18) from the corresponding update equations over particles defined as ground assignments (equation (19.10)). Your update rules must sum over all assignments consistent with the partition.

**Exercise 19.30**

Consider the distribution over partitions induced by the Chinese restaurant process.

- Find a closed-form formula for the probability induced by this process for any partition  $\sigma$  of the guests. Show that this probability is invariant to the order the guests enter the restaurant.
- Show that a Gibbs sampling process over the partitions generated by this algorithm satisfies equation (19.19) and equation (19.20).