

Algorithm 15.1 Filtering in a DBN using a template clique tree

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Procedure CTree-Filter-DBN (
     $\langle \mathcal{B}_0, \mathcal{B}_\rightarrow \rangle$ , // DBN
     $\mathbf{o}^{(1)}, \mathbf{o}^{(2)}, \dots$  // Observation sequence
)
1 Construct template clique tree  $\Upsilon$  over  $\mathcal{X}_I \cup \mathcal{X}'$ 
2  $\sigma^{(0)} \leftarrow P_{\mathcal{B}_0}(\mathcal{X}_I^{(0)})$ 
3 for  $t = 1, 2, \dots$ 
4    $\mathcal{T}^{(t)} \leftarrow \Upsilon$ 
5   Multiply  $\sigma^{(t-1)}(\mathcal{X}_I^{(t-1)})$  into  $\mathcal{T}^{(t)}$ 
6   Instantiate  $\mathcal{T}^{(t)}$  with  $\mathbf{o}^{(t)}$ 
7   Calibrate  $\mathcal{T}^{(t)}$  using clique tree inference
8   Extract  $\sigma^{(t)}(\mathcal{X}_I^{(t)})$  by marginalization

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it would require an exponential number of entries in the joint. At first glance, this argument appears specious. After all, one of the key benefits of graphical models is that high-dimensional distributions can be represented compactly by using factorization. It certainly appears plausible that we should be able to find a compact representation for our belief state and use our structured inference algorithms to manipulate it efficiently. As we now show, this very plausible impression turns out to be false.

Example 15.2

Consider our car network of figure 6.1, and consider our belief state at some time t . Intuitively, it seems as if there should be some conditional independence relations that hold in this network. For example, it seems as if $\text{Weather}^{(2)}$ and $\text{Location}^{(2)}$ should be uncorrelated. Unfortunately, they are not: if we examine the unrolled DBN, we see that there is an active trail between them going through $\text{Velocity}^{(1)}$ and $\text{Weather}^{(0)}$, $\text{Weather}^{(1)}$. This path is not blocked by any of the time 2 variables; in particular, $\text{Weather}^{(2)}$ and $\text{Location}^{(2)}$ are not conditionally independent given $\text{Velocity}^{(2)}$. In general, a similar analysis can be used to show that, for $t \geq 2$, no conditional independence assumptions hold in $\sigma^{(t)}$. ■

entanglement

This phenomenon, known as *entanglement*, has significant implications. As we discussed, there is a direct relationship between conditional independence properties of a distribution and our ability to represent it as a product of factors. Thus, a distribution that has no independence properties does not admit a compact representation in a factored form.

Unfortunately, the entanglement phenomenon is not specific to this example. Indeed, it holds for a very broad class of DBNs. We demonstrate it for a large subclass of DBNs that exhibit a very regular structure. We begin by introducing a few useful concepts.

Definition 15.1

persistent
independence

For a DBN over \mathcal{X} , and $\mathbf{X}, \mathbf{Y}, \mathbf{Z} \subset \mathcal{X}$, we say that the independence $(\mathbf{X} \perp \mathbf{Y} \mid \mathbf{Z})$ is persistent if $(\mathbf{X}^{(t)} \perp \mathbf{Y}^{(t)} \mid \mathbf{Z}^{(t)})$ holds for every t . ■

Persistent independencies are independence properties of the belief state, and are therefore precisely what we need in order to provide a time-invariant factorization of the belief state.