Algorithm 13.5 Alpha-expansion algorithm

Procedure Alpha-Expansion (
  ϵ, // Singleton and pairwise energies
  $\boldsymbol{x}$ // Some initial assignment
)
1 **repeat**
2 change $\leftarrow$ *false*
3 **for** $k = 1, \ldots, K$
4 $\boldsymbol{t} \leftarrow$ Alpha-Expand$(\epsilon, \boldsymbol{x}, v_k)$
5 **for** $i = 1, \ldots, n$
6 **if** $t_i = 1$ **then**
7 $x_i \leftarrow v_k$ // If $t_i = 0$, x_i doesn't change
8 change $\leftarrow$ *true*
9 **until** change $=$ *false*
10 **return** $(\boldsymbol{x})$

Procedure Alpha-Expand (
  ϵ,
  $\boldsymbol{x}$ // Current assignment
  v // Expansion label
)
1 Define ϵ' as in equation (13.34)
2 **return** MinCut-MAP(ϵ')

Box 13.B — Case Study: Energy Minimization in Computer Vision. *Over the past few years, MRFs have become a standard tool for addressing a range of low-level vision tasks, some of which we reviewed in box 4.B. As we discussed, the pairwise potentials in these models are often aimed at penalizing discrepancies between the values of adjacent pixels, and hence they often naturally satisfy the submodularity assumption that are necessary for the application of graph cut methods. Also very popular is the TRW-S variant of the convex belief propagation algorithms, described in box 13.A. Standard belief propagation has also been used in multiple applications.*

Vision problems pose some significant challenges. Although the grid structures associated with images are not dense, they are very large, and they contain many tight loops, which can pose difficulties for convergence of the message passing algorithm. Moreover, in some tasks, such as stereo reconstruction, *the value space of the variables is a discretization of a continuous space, and therefore many values are required to get a reasonable approximation. As a consequence, the representation of the pairwise potentials can get very large, leading to memory problems.*

stereo reconstruction

A number of fairly comprehensive empirical studies have been done comparing the various methods on a suite of computer-vision benchmark problems. By and large, it seems that for the grid-structured networks that we described, graph-cut methods with the alpha-expansion step and TRW-S are fairly comparable, with the graph-cut methods dominating in running time; both significantly