**Algorithm 12.5 Generating a Markov chain trajectory**

**Procedure** MCMC-Sample (
  $P^{(0)}(\boldsymbol{X})$, // Initial state distribution
  $\mathcal{T}$, // Markov chain transition model
  $T$ // Number of time steps
)
  Sample $\boldsymbol{x}^{(0)}$ from $P^{(0)}(\boldsymbol{X})$
  **for** $t = 1, \ldots, T$
    Sample $\boldsymbol{x}^{(t)}$ from $\mathcal{T}(\boldsymbol{x}^{(t-1)} \to \boldsymbol{X})$
    **return** $\boldsymbol{x}^{(0)}, \ldots, \boldsymbol{x}^{(T)}$

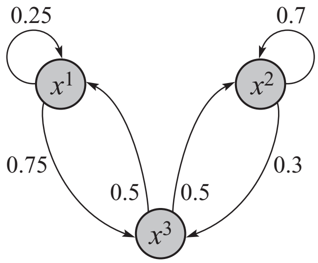


**Figure 12.4 A simple Markov chain**

### 12.3.2.3 Stationary Distributions

Intuitively, as the process converges, we would expect $P^{(t+1)}$ to be close to $P^{(t)}$. Using equation (12.20), we obtain:

$$P^{(t)}(\boldsymbol{x}') \approx P^{(t+1)}(\boldsymbol{x}') = \sum_{\boldsymbol{x} \in Val(\boldsymbol{X})} P^{(t)}(\boldsymbol{x})\mathcal{T}(\boldsymbol{x} \to \boldsymbol{x}').$$

At convergence, we would expect the resulting distribution $\pi(\boldsymbol{X})$ to be an equilibrium relative to the transition model; that is, the probability of being in a state is the same as the probability of transitioning into it from a randomly sampled predecessor. Formally:

**Definition 12.3**

stationary distribution

*A distribution* $\pi(\boldsymbol{X})$ *is a* stationary distribution *for a Markov chain* $\mathcal{T}$ *if it satisfies:*

$$\pi(\boldsymbol{X} = \boldsymbol{x}') = \sum_{\boldsymbol{x} \in Val(\boldsymbol{X})} \pi(\boldsymbol{X} = \boldsymbol{x})\mathcal{T}(\boldsymbol{x} \to \boldsymbol{x}'). \tag{12.21}$$

■

A stationary distribution is also called an *invariant distribution*.[2]

2. If we view the transition model as a matrix defined as $A_{i,j} = \mathcal{T}(\boldsymbol{x}_i \to \boldsymbol{x}_j)$, then a stationary distribution is an eigen-vector of the matrix, corresponding to the eigen-value 1. In general, many aspects of the theory of Markov chains have an algebraic interpretation in terms of matrices and vectors.