12.3. Markov Chain Monte Carlo Methods

Algorithm 12.5 Generating a Markov chain trajectory

Procedure MCMC-Sample (  
  \(P^{(0)}(X)\), // Initial state distribution  
  \(T\), // Markov chain transition model  
  \(T\) // Number of time steps  
)

1. Sample \(x^{(0)}\) from \(P^{(0)}(X)\)
2. for \(t = 1, \ldots, T\)
3. Sample \(x^{(t)}\) from \(T(x^{(t-1)} \rightarrow X)\)
4. return \(x^{(0)}, \ldots, x^{(T)}\)

![Figure 12.4 A simple Markov chain](image)

12.3.2.3 Stationary Distributions

Intuitively, as the process converges, we would expect \(P^{(t+1)}\) to be close to \(P^{(t)}\). Using equation (12.20), we obtain:

\[
P^{(t)}(x') \approx P^{(t+1)}(x') = \sum_{x \in \text{Val}(X)} P^{(t)}(x)T(x \rightarrow x').
\]

At convergence, we would expect the resulting distribution \(\pi(X)\) to be an equilibrium relative to the transition model; that is, the probability of being in a state is the same as the probability of transitioning into it from a randomly sampled predecessor. Formally:

**Definition 12.3**

A distribution \(\pi(X)\) is a stationary distribution for a Markov chain \(T\) if it satisfies:

\[
\pi(X = x') = \sum_{x \in \text{Val}(X)} \pi(X = x)T(x \rightarrow x').
\]  \(\text{(12.21)}\)

A stationary distribution is also called an *invariant distribution*.

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2. If we view the transition model as a matrix defined as \(A_{i,j} = T(x_i \rightarrow x_j)\), then a stationary distribution is an eigen-vector of the matrix, corresponding to the eigen-value 1. In general, many aspects of the theory of Markov chains have an algebraic interpretation in terms of matrices and vectors.