Algorithm 11.5 Modified version of BU-Message **that incorporates message projection**

```
Procedure EP-Message (
   i,   // sending clique
   j    // receiving clique
 )
  σ̃_{i→j} ← M-project-distr_{Q_{i,j}}(φ⃗_i)
     // marginalize and project the clique over the sepset
  Remove old δ̃_{i→j} from φ⃗_j
  δ̃_{i→j} ← σ̃_{i→j} / δ̃_{j→i}
     // divide by the message from from C_j to C_i
  Insert new δ̃_{i→j} into φ⃗_j
```

keep the previous message sent over the edge, but rather keep the messages $\tilde{\delta}_{i \to j}$ sent in both directions; this more refined bookkeeping is necessary for dividing by the correct message following the approximation. This algorithm is called *expectation propagation* (EP) for reasons that are explained later.

expectation propagation

Example 11.8

To understand the behavior of this algorithm, consider the application of belief-update message propagation to example 11.7. Suppose we initialize all messages to 1 *and use updates of the form equation (11.39). We start by propagating a message* $\tilde{\sigma}_{1\to 2} = \text{M-project-distr}_{1,2}(\tilde{\beta}_1)$ *from* $\boldsymbol{C}_1$ *to* $\boldsymbol{C}_2$*. Because* $\tilde{\delta}_{1\to 2}$ *at this stage is 1, the resulting update is exactly the one we discussed in example 11.7. If we now perform propagation from* $\boldsymbol{C}_2$ *to* $\boldsymbol{C}_1$*, we get the message* $\tilde{\delta}_{2\to 1}$ *derived in example 11.7, multiplied by a constant (since* $\tilde{\delta}_{1\to 2}$ *is uniform). At this point, as we discussed, the clique beliefs* β_1 *are a fairly reasonable approximation to the posterior.*

Using the revised update rule, we now project $\tilde{\beta}_1$*, and then divide by* $\tilde{\delta}_{2\to 1}$*:*

$$\tilde{\delta}_{1\to 2} \leftarrow \frac{\text{M-project-distr}_{1,2}(\tilde{\beta}_1)}{\tilde{\delta}_{2\to 1}}.$$

This quotient, which is thensubsequently used to update $\boldsymbol{C}_2$*, is very different from the previous update* $\tilde{\delta}_{2\to 1}$*. Specifically, The marginal* $\tilde{\beta}_1$ *at this stage puts a posterior of* $0.642 + 0.031 = 0.673$ *on* a^1*, and* 0.642 *on* b^1*. To avoid double-counting the contribution of* $\tilde{\delta}_{2\to 1}$*, we need to divide this marginal by this message. After we normalize messages, we obtain:*

$$\begin{aligned}
\tilde{\delta}_{1\to 2}[a^1] &\leftarrow \frac{\frac{0.673}{0.904}}{\frac{0.673}{0.904} + \frac{0.327}{0.086}} = \frac{0.744}{4.15} = 0.179 \\
\tilde{\delta}_{1\to 2}[a^0] &\leftarrow \frac{\frac{0.327}{0.086}}{\frac{0.673}{0.904} + \frac{0.327}{0.086}} = \frac{3.406}{4.15} = 0.821 \\
\tilde{\delta}_{1\to 2}[b^1] &\leftarrow \frac{\frac{0.642}{0.173}}{\frac{0.642}{0.173} + \frac{0.358}{0.827}} = \frac{3.710}{4.413} = 0.895 \\
\tilde{\delta}_{1\to 2}[b^0] &\leftarrow \frac{\frac{0.358}{0.827}}{\frac{0.642}{0.173} + \frac{0.358}{0.827}} = \frac{0.433}{4.144} = 0.105.
\end{aligned}$$