edge $(i–j)$, connecting the clusters $\boldsymbol{C}_i$ and $\boldsymbol{C}_j$, we have that

$$\sum_{\boldsymbol{C}_i - \boldsymbol{S}_{i,j}} \beta_i = \sum_{\boldsymbol{C}_j - \boldsymbol{S}_{i,j}} \beta_j;$$

that is, the two clusters agree on the marginal of variables in $\boldsymbol{S}_{i,j}$. Note that this definition is weaker than cluster tree calibration, since the clusters do not necessarily agree on the joint marginal of all the variables they have in common, but only on those variables in the sepset. However, if a calibrated cluster graph satisfies the running intersection property, then the marginal of a variable X is identical in all the clusters that contain it.

Algorithm 11.1 Calibration using sum-product belief propagation in a cluster graph

Procedure CGraph-SP-Calibrate (
  Φ, // Set of factors
  $\mathcal{U}$ // Generalized cluster graph Φ
)
  Initialize-CGraph
  while graph is not calibrated
    Select $(i–j) \in \mathcal{E}_{\mathcal{U}}$
    $\delta_{i\to j}(\boldsymbol{S}_{i,j}) \leftarrow$ SP-Message(i, j)
  for each clique i
    $\beta_i \leftarrow \psi_i \cdot \prod_{k \in \mathrm{Nb}_i} \delta_{k\to i}$
  return $\{\beta_i\}$

Procedure Initialize-CGraph (
  $\mathcal{U}$
)
  for each cluster $\boldsymbol{C}_i$
    $\beta_i \leftarrow \prod_{\phi\,:\,\alpha(\phi)=i} \phi$
  for each edge $(i–j) \in \mathcal{E}_{\mathcal{U}}$
    $\delta_{i\to j} \leftarrow 1$
    $\delta_{j\to i} \leftarrow 1$

Procedure SP-Message (
  i, // sending clique
  j // receiving clique
)
  $\psi(\boldsymbol{C}_i) \leftarrow \psi_i \cdot \prod_{k \in (\mathrm{Nb}_i - \{j\})} \delta_{k\to i}$
  $\tau(\boldsymbol{S}_{i,j}) \leftarrow \sum_{\boldsymbol{C}_i - \boldsymbol{S}_{i,j}} \psi(\boldsymbol{C}_i)$
  return $\tau(\boldsymbol{S}_{i,j})$

How do we calibrate a cluster graph? Because calibration is a local property that relates adjoining clusters, we want to try to ensure that each cluster is sharing information with its