Algorithm 9.6 Rule splitting algorithm

```plaintext
Procedure Rule-Split (ρ)
    ρ = (c; p),  // Rule to be split
    c'  // Context to split on
1    if c ∼ c' then return ρ
2    if Scope[c] ⊆ Scope[c'] then return ρ
3    Select Y ∈ Scope[c'] − Scope[c]
4    R ← Split(ρ ∩ Y)
5    R' ← ∪ ρ'' ∈ R Rule-Split(ρ'', c')
6    return R'
```

The rules ρ3 on the one hand, and ρ7, ρ8 on the other, have compatible contexts, so we can choose to combine them. We begin by splitting ρ3 and ρ7 on each other’s context, which results in:

\[
\begin{align*}
ρ_{15} &= \langle a^0, b^1, d^0, e^0; 1 − q_2 \rangle \\
ρ_{16} &= \langle a^0, b^1, d^0, e^1; 1 − q_2 \rangle \\
ρ_{17} &= \langle a^0, b^0, d^0, e^0; 1 − p_1 \rangle \\
ρ_{18} &= \langle a^0, b^1, d^0, e^0; 1 − p_1 \rangle
\end{align*}
\]

The contexts of ρ15 and ρ18 match, so we can now apply rule product, replacing the pair by:

\[
\begin{align*}
ρ_{19} &= \langle a^0, b^1, d^0, e^0; (1 − q_2)(1 − p_1) \rangle
\end{align*}
\]

We can now split ρ8 using the context of ρ16 and multiply the matching rules together, obtaining

\[
\begin{align*}
ρ_{20} &= \langle a^0, b^0, d^0, e^1; p_1 \rangle \\
ρ_{21} &= \langle a^0, b^1, d^0, e^1; (1 − q_2)p_1 \rangle
\end{align*}
\]

The resulting rule set contains ρ17, ρ19, ρ20, ρ21 in place of ρ3, ρ7, ρ8.

We can apply a similar process to ρ4 and ρ9, ρ10, which leads to their substitution by the rule set:

\[
\begin{align*}
ρ_{22} &= \langle a^0, b^0, d^1, e^0; 1 − p_2 \rangle \\
ρ_{23} &= \langle a^0, b^1, d^1, e^0; q_2(1 − p_2) \rangle \\
ρ_{24} &= \langle a^0, b^0, d^1, e^1; p_2 \rangle \\
ρ_{25} &= \langle a^0, b^1, d^1, e^1; q_2p_2 \rangle
\end{align*}
\]

We can now eliminate D in the context a^0, b^1, e^1. The only rules in R^+ compatible with this context are ρ21 and ρ25. We extract them from R^+ and sum them; the resulting rule \langle a^0, b^1, e^1; (1 − q_2)p_1 + q_2p_2 \rangle, is then inserted into R^−. We can similarly eliminate D in the context a^0, b^1, e^0.

The process continues, with rules being split and multiplied. When D has been eliminated in a set of mutually exclusive and exhaustive contexts, then we have exhausted all rules involving D; at this point, R^+ is empty, and the process of eliminating D terminates.

\[ \blacksquare \]