Algorithm 9.6 Rule splitting algorithm

Procedure Rule-Split (
  $\rho = \langle \boldsymbol{c}; p \rangle$, // Rule to be split
  $\boldsymbol{c}'$ // Context to split on
)
  if $\boldsymbol{c} \not\sim \boldsymbol{c}'$ **then return** ρ
  if $\mathit{Scope}[\boldsymbol{c}] \subseteq \mathit{Scope}[\boldsymbol{c}']$ **then return** ρ
  Select $Y \in \mathit{Scope}[\boldsymbol{c}'] - \mathit{Scope}[\boldsymbol{c}]$
  $\mathcal{R} \leftarrow \mathit{Split}(\rho \angle Y)$
  $\mathcal{R}' \leftarrow \cup_{\rho'' \in \mathcal{R}}$ Rule-Split$(\rho'', \boldsymbol{c}')$
  return $\mathcal{R}'$

The rules ρ_3 on the one hand, and ρ_7, ρ_8 on the other, have compatible contexts, so we can choose to combine them. We begin by splitting ρ_3 and ρ_7 on each other's context, which results in:

$$\left\{ \begin{array}{ll} \rho_{15} & \langle a^0, b^1, d^0, e^0; 1 - q_2 \rangle \\ \rho_{16} & \langle a^0, b^1, d^0, e^1; 1 - q_2 \rangle \\ & \\ \rho_{17} & \langle a^0, b^0, d^0, e^0; 1 - p_1 \rangle \\ \rho_{18} & \langle a^0, b^1, d^0, e^0; 1 - p_1 \rangle \end{array} \right\}$$

The contexts of ρ_{15} and $\rho 18$ match, so we can now apply rule product, replacing the pair by:

$$\left\{ \ \rho_{19} \quad \langle a^0, b^1, d^0, e^0; (1 - q_2)(1 - p_1) \rangle \ \right\}$$

We can now split ρ_8 using the context of ρ_{16} and multiply the matching rules together, obtaining

$$\left\{ \begin{array}{lc} \rho_{20} & \langle a^0, b^0, d^0, e^1; p_1 \rangle \\ \rho_{21} & \langle a^0, b^1, d^0, e^1; (1 - q_2) p_1 \rangle \end{array} \right\}.$$

The resulting rule set contains $\rho_{17}, \rho_{19}, \rho_{20}, \rho_{21}$ in place of ρ_3, ρ_7, ρ_8.

We can apply a similar process to ρ_4 and ρ_9, ρ_{10}, which leads to their substitution by the rule set:

$$\left\{ \begin{array}{lc} \rho_{22} & \langle a^0, b^0, d^1, e^0; 1 - p_2 \rangle \\ \rho_{23} & \langle a^0, b^1, d^1, e^0; q_2(1 - p_2) \rangle \\ \rho_{24} & \langle a^0, b^0, d^1, e^1; p_2 \rangle \\ \rho_{25} & \langle a^0, b^1, d^1, e^1; q_2 p_2 \rangle \end{array} \right\}.$$

We can now eliminate D in the context a^0, b^1, e^1. The only rules in $\mathcal{R}^+$ compatible with this context are ρ_{21} and ρ_{25}. We extract them from $\mathcal{R}^+$ and sum them; the resulting rule $\langle a^0, b^1, e^1; (1 - q_2)p_1 + q_2 p_2 \rangle$, is then inserted into $\mathcal{R}^-$. We can similarly eliminate D in the context a^0, b^1, e^0.

The process continues, with rules being split and multiplied. When D has been eliminated in a set of mutually exclusive and exhaustive contexts, then we have exhausted all rules involving D; at this point, $\mathcal{R}^+$ is empty, and the process of eliminating D terminates. ■