5.3. Context-Specific CPDs

Figure 5.7  The graph of figure 5.3, after we remove spurious edges: (a) in the context \( A = a^0 \); (b) in the context \( S = s^1 \).

Can we capture this intuition formally? Consider the dependence structure in the context \( A = a^0 \). Intuitively, in this context, the edges \( S \rightarrow J \) and \( L \rightarrow J \) are both redundant, since we know that \((J \perp_{c} S, L \mid a^0)\). Thus, our intuition is that we should check for d-separation in the graph without this edge. Indeed, we can show that this is a sound check for CSI conditions.

**Definition 5.7**

Let \( P(X \mid Pa_X) \) be a CPD, let \( Y \in Pa_X \), and let \( c \) be a context. We say that the edge \( Y \rightarrow X \) is spurious in the context \( c \) if \( P(X \mid Pa_X) \) satisfies \((X \perp_{c} Y \mid Pa_X \setminus \{Y\}, c')\), where \( c' = c\langle Pa_X \rangle \) is the restriction of \( c \) to variables in \( Pa_X \).

If we represent CPDs with rules, then we can determine whether an edge is spurious by examining the reduced rule set. Let \( R \) be the rule-based CPD for \( P(X \mid Pa_X) \), then the edge \( Y \rightarrow X \) is spurious in context \( c \) if \( Y \) does not appear in the reduced rule set \( R[c] \).

**Algorithm 5.2 Computing d-separation in the presence of context-specific CPDs**

```
Procedure CSI-sep (G, c, X, Y, Z)
    G' ← G
    for each edge Y → X in G'
        if Y → X is spurious given c in G then
            Remove Y → X in G'
    return d-sep\(_{G'}\)(X; Y | Z, c)
```

Now we can define **CSI-separation**, a variant of d-separation that takes CSI into account. This notion, defined procedurally in algorithm 5.2, is straightforward: we use local considerations to remove spurious edges and then apply standard d-separation to the resulting graph. We say that

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**Figure 5.7**

The graph of figure 5.3, after we remove spurious edges: (a) in the context \( A = a^0 \); (b) in the context \( S = s^1 \).