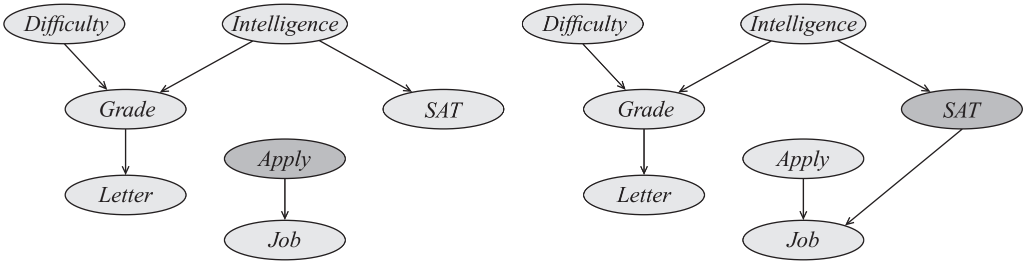

(a) (b)

Figure 5.7 The graph of figure 5.3, after we remove spurious edges: (a) in the context $A = a^0$; (b) in the context $S = s^1$.

Can we capture this intuition formally? Consider the dependence structure in the context $A = a^0$. Intuitively, in this context, the edges $S \rightarrow J$ and $L \rightarrow J$ are both redundant, since we know that $(J \perp_c S, L \mid a^0)$. Thus, our intuition is that we should check for d-separation in the graph without this edge. Indeed, we can show that this is a sound check for CSI conditions.

Definition 5.7

spurious edge

Let $P(X \mid \mathrm{Pa}_X)$ be a CPD, let $Y \in \mathrm{Pa}_X$, and let $\boldsymbol{c}$ be a context. We say that the edge $Y \rightarrow X$ is spurious *in the context $\boldsymbol{c}$ if $P(X \mid \mathrm{Pa}_X)$ satisfies $(X \perp_c Y \mid \mathrm{Pa}_X - \{Y\}, \boldsymbol{c}')$, where $\boldsymbol{c}' = \boldsymbol{c}\langle \mathrm{Pa}_X \rangle$ is the restriction of $\boldsymbol{c}$ to variables in Pa_X.* ■

If we represent CPDs with rules, then we can determine whether an edge is spurious by examining the reduced rule set. Let $\mathcal{R}$ be the rule-based CPD for $P(X \mid \mathrm{Pa}_X)$, then the edge $Y \rightarrow X$ is spurious in context $\boldsymbol{c}$ if Y does not appear in the reduced rule set $\mathcal{R}[\boldsymbol{c}]$.

Algorithm 5.2 Computing d-separation in the presence of context-specific CPDs

Procedure CSI-sep (
  $\mathcal{G}$, // Bayesian network structure
  $\boldsymbol{c}$, // Context
  $\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{Z}$ // Is $\boldsymbol{X}$ CSI-separated from $\boldsymbol{Y}$ given $\boldsymbol{Z}, \boldsymbol{c}$
)
  $\mathcal{G}' \leftarrow \mathcal{G}$
  for each edge $Y \rightarrow X$ in $\mathcal{G}'$
    if $Y \rightarrow X$ is spurious given $\boldsymbol{c}$ in $\mathcal{G}$ **then**
      Remove $Y \rightarrow X$ in $\mathcal{G}'$
  return $d\text{-}sep_{\mathcal{G}'}(\boldsymbol{X}; \boldsymbol{Y} \mid \boldsymbol{Z}, \boldsymbol{c})$

CSI-separation

Now we can define *CSI-separation*, a variant of d-separation that takes CSI into account. This notion, defined procedurally in algorithm 5.2, is straightforward: we use local considerations to remove spurious edges and then apply standard d-separation to the resulting graph. We say that