Let $\sigma_M = \sqrt{\mathbb{V}ar[T_{\mathcal{D}}]}$ be the standard deviation of $T_{\mathcal{D}}$ for $\mathcal{D}$ of size $M$. Using the multiplicative Chernoff bound, we can show that

$$P_{\mathcal{D}}(|T_{\mathcal{D}} - p| \geq k\sigma) \leq 2e^{-k^2/6}. \tag{A.2}$$

This inequality should be contrasted with the Chebyshev inequality. The big difference owes to the fact that the Chernoff bound exploits the particular properties of the distribution of $T_{\mathcal{D}}$.

## A.3 Algorithms and Algorithmic Complexity

In this section, we briefly review relevant algorithms and notions from algorithmic complexity. Cormen et al. (2001) is a good source for learning about algorithms, data structures, graph algorithms, and algorithmic complexity; Papadimitriou (1993) and Sipser (2005) provide a good introduction to the key concepts in computational complexity.

### A.3.1 Basic Graph Algorithms

Given a graph structure, there are many useful operations that we might want to perform. For example, we might want to determine whether there is a certain type of path between two nodes. In this section, we survey algorithms for performing two key tasks that will be of use in several places throughout this book. Additional algorithms, for more specific tasks, are presented as they become relevant.

**Algorithm A.1 Topological sort of a graph**

**Procedure** Topological-Sort (
  $\mathcal{G} = (\mathcal{X}, \mathcal{E})$ // A directed graph
)
  Set all nodes to be unmarked
  **for** $i = 1, \ldots, n$
    Select any unmarked node $X$ all of whose parents are marked
    $d(X) \leftarrow i$
    Mark $X$
  **return** $(\vec{d})$

topological ordering

One algorithm, shown in algorithm A.1, finds a *topological ordering* of the nodes in the graph, as defined in definition 2.19.

maximum weight spanning tree

Another useful algorithm is one that finds, in a weighted undirected graph $\mathcal{H}$ with nonnegative edge weights, a *maximum weight spanning tree*. More precisely, a subgraph is said to be a *spanning tree* if it is a tree and it spans all vertices in the graph. Similarly, a *spanning forest* is a forest that spans all vertices in the graph. A maximum weight spanning tree (or forest) is the tree (forest) whose edge-weight sum is largest among all spanning trees (forests).